

Manifestation of spin-orbit interaction in tunneling between 2D electron layers

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An influence of spin-orbit interaction on the tunneling between two 2D electron layers is considered. Particular attention is addressed to the relation between the contribution of Rashba and Dresselhaus types. It is shown that without scattering of the electrons, the tunneling conductance can either exhibit resonances at certain voltage values or be substantially suppressed over the whole voltage range. The dependence of the conductance on voltage turns out to be very sensitive to the relation between Rashba and Dresselhaus contributions even in the absence of magnetic field. The elastic scattering broadens the resonances in the first case and restores the conductance to a larger magnitude in the latter one. These effects open possibility to determine the parameters of spin-orbit interaction and electrons scattering time in tunneling experiments with no necessity of external magnetic field.

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I. INTRODUCTION

Spin-orbit interaction (SOI) plays an important role in the widely studied spin-related effects and spintronic devices. In the latter it can be either directly utilized to create spatial separation of the spin-polarized charge carries or indirectly influence the device performance through spin-decoherence time. In 2D structures two kinds of SOI are known to be of the most importance, namely Rashba and Dresselhaus mechanisms. The first one characterized by parameter α is due to the structure inversion asymmetry (SIA) while the second one characterized by β is due to the bulk inversion asymmetry (BIA). Most brightly both of the contributions reveal themselves when the values of α and β are comparable. In this case a number of interesting effects occur: the electron energy spectrum becomes strongly anisotropic¹, the electron spin relaxation rate becomes dependent on the spin orientation in the plane of the quantum well², a magnetic break-down should be observed in the Shubnikov de Haas effect³. The energy spectra splitting due to SOI can be observed in rather well-developed experiments as that based on Shubnikov–de Haas effect. However, these experiments can hardly tell about the partial contributions of the two mechanisms leaving the determination of the relation between α and β to be a more challenging task. At the same time, in some important cases spin relaxation time τ_s and spin polarization strongly depend on the $\frac{\alpha}{\beta}$ ratio. In this paper we consider the tunneling between 2D electron layers, which turns out to be sensitive to the relation between Rashba and Dresselhaus contributions. The specific feature of the tunneling in the system under consideration is that the energy and in-plane momentum conservation put tight restrictions on the tunneling. Without SOI the tunneling conductance exhibits delta function-like maximum at zero bias broadened by elastic scattering in the layers⁴, and fluctuations of the layers width⁵. Such a behavior was indeed observed in a number of experiments^{6,7,8}. Spin-orbit interaction splits the electron spectra into two subbands in

each layer. At that energy and momentum conservation can be fulfilled for the tunneling between opposite subbands of the layers at a finite voltage corresponding to the subbands splitting. However, if the parameters of SOI are equal for left and right layers, the tunneling remains prohibited due to orthogonality of the appropriate spinor eigenstates. In⁹ it was pointed out that this restriction can also be eliminated if Rashba parameters are different for the two layers. A structure design was proposed¹⁰ where exactly opposite values of the Rashba parameters result from the built-in electric field in the left layer being opposite to that in the right layer. Because the SOI of Rashba type is proportional to the electric field, this would result in $\alpha^R = -\alpha^L$, where α^L and α^R are the Rashba parameters for the left and right layers respectively. In this case the peak of the conductance should occur at the voltage U_0 corresponding to the energy of SOI: $eU_0 = \pm 2\alpha k_F$, where k_F is Fermi wavevector. In this paper we consider arbitrary Rashba and Dresselhaus contributions and show how qualitatively different situations can be realized depending on their partial impact. In some cases the structure of the electrons eigenstates suppresses tunneling at ever voltage. At that the scattering is important as it restores the features of voltage-current characteristic containing information about SOI parameters. Finally the parameters α and β can be obtained in the tunneling experiment which unlike other spin-related experiments requires neither magnetic field nor polarized light.

II. CALCULATIONS

We consider two 2D electron layers separated by potential barrier at zero temperature (see Fig.1). We shall consider only one level of size quantization and not too narrow barrier so that the electrons wavefunctions in the left and right layers overlap weakly. The system can be described by the phenomenological tunneling Hamiltonian^{4,5,11}:

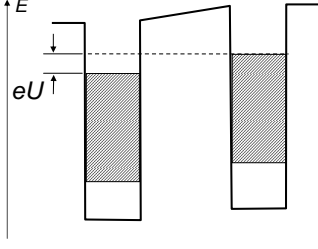


FIG. 1: Energy diagram of two 2D electron layers.

$$H = H_0^L + H_0^R + H_T, \quad (1)$$

where H_0^L, H_0^R are the partial Hamiltonians for the left and right layers respectively, H_T is the tunneling term. With account for the elastic scattering and SOI in the layers the partial Hamiltonians and the tunneling term have the following form in representation of secondary quantization:

$$\begin{aligned} H_0^l &= \sum_{k,\sigma} \varepsilon_k^l c_{k\sigma}^{l+} c_{k\sigma}^l + \sum_{k,k',\sigma} V_{kk'}^l c_{k\sigma}^{l+} c_{k'\sigma}^l + H_{SO}^l \\ H_T &= \sum_{k,k',\sigma,\sigma'} T_{kk'\sigma\sigma'} (c_{k\sigma}^{L+} c_{k'\sigma'}^R + c_{k'\sigma'}^{R+} c_{k\sigma}^L), \end{aligned} \quad (2)$$

Here index l is used for the layer designation and can take the values $l = R$ for the right layer, $l = L$ for the left layer. By k here and further throughout the paper we denote the wavevector aligned parallel to the layers planes, σ denotes spin polarization and can take the values $\sigma = \pm 1/2$. ε_k^l is the energy of an electron in the layer l having in-plane wavevector k . It can be expressed as:

$$\varepsilon_k^l = \varepsilon + \varepsilon_0^l + \Delta^l, \quad (3)$$

where $\varepsilon = \frac{\hbar^2 k^2}{2m}$, m being electron's effective mass, ε_0^l and Δ^l are the size quantization energy and the energy shift due to external voltage for the layer l . We shall also use the value $\Delta^{ll'}$ defined as $\Delta^{ll'} = (\Delta^l - \Delta^{l'}) + (\varepsilon_0^l - \varepsilon_0^{l'})$. Similar notation will be used for spin polarization denoted by indices σ, σ' . The second term in the Hamiltonian (2) $V_{kk'}^l$ is the matrix element of the scattering operator. We consider only elastic scattering. The tunneling term H_T in (2) is described by the tunneling constant $T_{kk'\sigma\sigma'}$, which has the meaning of size quantization levels splitting due to the wavefunctions overlap. By lowercase t we shall denote the overlap integral itself. Our consideration is valid only for the case of weak overlapping, i.e. $t \ll 1$. Parametrically $T \sim t\varepsilon_F$, where ε_F is the electrons Fermi energy. The term H_{SO}^l describes the spin-orbit part of the Hamiltonian:

$$\hat{H}_{SO}^l = \alpha^l (\boldsymbol{\sigma} \times \mathbf{k})_z + \beta^l (\sigma_x k_x - \sigma_y k_y), \quad (4)$$

where σ_i are the Pauli matrices, α^l, β^l are respectively the parameters of Rashba and Dresselhaus interactions for the layer l . In the secondary quantization representation:

$$\begin{aligned} \hat{H}_{SO}^l &= \alpha^l \sum_k (k_y - ik_x) c_{k\sigma}^{l+} c_{k\sigma'}^l + (k_y + ik_x) c_{k\sigma'}^{l+} c_{k\sigma}^l \\ &+ \beta^l \sum_k (k_x - ik_y) c_{k\sigma}^{l+} c_{k\sigma'}^l + (k_x + ik_y) c_{k\sigma'}^{l+} c_{k\sigma}^l \end{aligned} \quad (5)$$

The operator of the tunneling current can be expressed as⁴:

$$\hat{I} = \frac{ie}{\hbar} \sum_{k,k',\sigma,\sigma'} T_{kk'\sigma\sigma'} (\hat{\rho}_{kk'\sigma\sigma'}^{RL} - \hat{\rho}_{k'k\sigma'\sigma}^{LR}), \quad (6)$$

where $\hat{\rho}_{kk'\sigma\sigma'}^{ll'} = c_{k,\sigma}^{l+} c_{k',\sigma'}^{l'}$. We shall assume the case of in-plane momentum and the spin projection being conserved in the tunneling event so the tunneling constant $T_{kk'\sigma\sigma'}$ has the form $T_{kk'\sigma\sigma'} = T \delta_{kk'} \delta_{\sigma\sigma'}$, where δ is the Cronecker symbol. The tunneling current is then given by

$$I = \frac{ie}{\hbar} T \int dk \text{Tr} (\langle \hat{\rho}_{k\sigma}^{RL} \rangle - \langle \hat{\rho}_{k\sigma}^{LR} \rangle), \quad (7)$$

where $\langle \rangle$ denotes the expectation value in quantum-mechanical sense. For further calculations it is convenient to introduce vector operator $\hat{\mathbf{S}}_{kk'}^{ll'} = \{\hat{S}_0, \hat{\mathbf{s}}\} = \{\text{Tr}(\hat{\rho}_{kk'\sigma\sigma'}^{ll'}), \text{Tr}(\boldsymbol{\sigma} \hat{\rho}_{kk'\sigma\sigma'}^{ll'})\}$. This vector fully determines the current because the latter can be expressed through the difference $\hat{S}_{0k}^{RL} - \hat{S}_{0k}^{LR}$. The time evolution of $\hat{\mathbf{S}}_{kk'}^{ll'}$ is governed by:

$$\frac{d\hat{\mathbf{S}}_{kk'}^{ll'}}{dt} = \frac{i}{\hbar} [H, \hat{\mathbf{S}}_{kk'}^{ll'}] \quad (8)$$

In the standard way of reasoning¹³ we assume adiabatic onset of the interaction with characteristic time w^{-1} . We will set $w = 0$ in the final expression. With this (8) turns into:

$$(\hat{\mathbf{S}}_{kk'}^{ll'} - \hat{\mathbf{S}}_{kk'}^{(0)ll'})w = \frac{i}{\hbar} [H, \hat{\mathbf{S}}_{kk'}^{ll'}] \quad (9)$$

Here $\hat{\mathbf{S}}_{kk'}^{(0)ll'}$ represents the stationary solution of (8) without interaction. By interaction here we mean the tunneling and the elastic scattering by impurities but not the external voltage. The role of the latter is merely shifting the layers by eU on the energy scale. From such defined interaction it immediately follows that the only non-zero elements of $\hat{\mathbf{S}}_{kk'}^{(0)ll'}$ are that with $l = l'$ and $k = k'$. In further abbreviations we will avoid duplication of the indices i.e. write single l instead of ll and k instead of kk :

$$\hat{\mathbf{S}}_{kk'}^{(0)ll'} = \hat{\mathbf{S}}_k^{(0)l} \delta_{kk'} \delta_{ll'} \quad (10)$$

With use of fermion commutation rules

$$\begin{aligned} \{c_i c_k\} &= \{c_i^+ c_k^+\} = 0 \\ \{c_i c_k^+\} &= \delta_{ik} \end{aligned}$$

the calculations performed in a way similar to¹³ bring us to the following system of equations with respect to $\hat{\mathbf{S}}_k^{ll'}$:

$$\begin{aligned} 0 &= (\Delta^{ll'} + i\hbar w) \hat{\mathbf{S}}_k^{ll'} + T (\hat{\mathbf{S}}_k^{l'} - \hat{\mathbf{S}}_k^l) + \mathbf{M}(k) \hat{\mathbf{S}}_k^{ll'} \\ &- \sum_{k'} \left(\frac{A_{kk'} \hat{\mathbf{S}}_k^{ll'} - B_{kk'} \hat{\mathbf{S}}_{k'}^{ll'}}{\varepsilon' - \varepsilon - \Delta^{ll'} + i\hbar w} + \frac{B_{kk'} \hat{\mathbf{S}}_k^{ll'} - A_{kk'} \hat{\mathbf{S}}_{k'}^{ll'}}{\varepsilon - \varepsilon' - \Delta^{ll'} + i\hbar w} \right) \end{aligned} \quad (11)$$

$$i\hbar w \left(\hat{\mathbf{S}}_k^{(0)l} - \hat{\mathbf{S}}_k^l \right) = T \left(\hat{\mathbf{S}}_k^{l'l} - \hat{\mathbf{S}}_k^{ll'} \right) + \mathbf{M}(k) \hat{\mathbf{S}}_k^l + \sum_{k'} \frac{2i\hbar w A_{kk'}}{(\varepsilon' - \varepsilon)^2 + (\hbar w)^2}, \quad (12)$$

where \mathbf{M} is a known matrix, depending on k and parameters of spin-orbit interaction in the layers. Here we also introduced the quadratic forms of the impurities potential matrix elements:

$$A_{kk'} \equiv |V_{k'k}^l|^2, \quad B_{kk'} \equiv V_{k'k}^l V_{kk'}^{l'} \quad (13)$$

As (11) and (12) comprise a system of linear integral equations these quantities enter the expression (7) for the current linearly and can be themselves averaged over spatial distribution of the impurities. In order to perform this averaging we assume the short range potential of impurities:

$$V(r) = \sum_a V_0 \delta(r - r_a) \quad (14)$$

The averaging immediately shows that the correlators $\langle A_{kk'} \rangle \equiv A$ and $\langle B_{kk'} \rangle \equiv B$ have different parametrical dependence on the tunneling transparency t , namely

$$\frac{B}{A} \sim t^2 \sim T^2 \quad (15)$$

We emphasize that this result holds for non-correlated distribution of the impurities as well as for their strongly correlated arrangement such as a thin layer of impurities placed in the middle of the barrier. The corresponding expressions for these two cases are given below. Index 'rand' stands for uniform impurities distribution and 'cor' for their correlated arrangement in the middle of the barrier ($z = 0$):

$$\begin{aligned} B^{rand} &= \frac{V_0^2 n}{W} \int dz f_l^2(z) f_{l'}^2(z) \sim \frac{V_0^2 n}{W} \frac{t^2}{d} \\ A^{rand} &= \frac{V_0^2 n}{W} \int dz f_l^4(z) \sim \frac{V_0^2 n}{W} \frac{1}{d} \\ B^{cor} &= \frac{V_0^2 n_s}{W} f_l^2(0) f_{l'}^2(0) \sim \frac{V_0^2 n_s}{W} \frac{t^2}{d} \\ A^{cor} &= \frac{V_0^2 n_s}{W} f_l^4(0) \sim \frac{V_0^2 n_s}{W} \frac{1}{d}, \end{aligned} \quad (16)$$

where n and n_s are bulk and surface concentrations of the impurities, W is the lateral area of the layers, d is the width of the barrier and $f(z)$ is the eigenfunction corresponding to the size quantization level, z is coordinate in the direction normal to the layers planes, $z = 0$ corresponding to the middle of the barrier¹⁰. Unlike¹⁰ and according to (15) we conclude that the correlator $\langle B_{kk'} \rangle$ has to be neglected as soon as we shall be interested in calculating the current within the order of T^2 . In the hereused method of calculation this result appears

quite naturally, however, it can be similarly traced in the technique used in¹⁰ (see Appendix). For the same reason the tunneling term should be dropped from (12) as it would give second order in T if (12) substituted into (11). According to (13) A can be expressed in terms of electrons scattering time:

$$\frac{1}{\tau} = \frac{2\pi}{\hbar} \nu \langle |V_{kk'}|^2 \rangle = \frac{2\pi}{\hbar} \nu A, \quad (17)$$

where ν is the 2D density of states $\nu = \frac{m}{2\pi\hbar^2}$. By means of Fourier transformation on energy variable the system (11),(12) can be reduced to the system of linear algebraic equations. Finally $\hat{\mathbf{S}}_k^{ll'}$ can be expressed as a function of $\hat{\mathbf{S}}_k^{(0)l}$. Consequently the current (7) becomes a function of $\langle \hat{\rho}_{k\sigma}^{(0)R} \rangle$, $\langle \hat{\rho}_{k\sigma}^{(0)L} \rangle$. For the considered case of zero temperature:

$$\langle \rho_{k\sigma}^{(0)l} \rangle = \frac{1}{2W} \theta(\varepsilon_F^l + \Delta^l - \varepsilon - \varepsilon_\sigma),$$

where

$$\varepsilon_\sigma = \pm |\alpha^l(k_x - ik_y) - \beta^l(ik_x - k_y)|,$$

. Without loss of generality we shall consider the case of identical layers and external voltage applied as shown in Fig.1:

$$\begin{aligned} \varepsilon_0^R &= \varepsilon_0^L \\ \Delta^L &= -\frac{eU}{2}, \Delta^R = +\frac{eU}{2} \\ \Delta^{RL} &= -\Delta^{LR} = eU \end{aligned}$$

The calculations can be simplified with account for two small parameters:

$$\begin{aligned} \xi &= \frac{\hbar}{\varepsilon_F \tau} \ll 1 \\ \eta &= \frac{eU}{\varepsilon_F} \ll 1 \end{aligned} \quad (18)$$

With (??) calculation yields the following expression for the current:

$$I = \frac{ie}{2\pi\hbar} T^2 \nu \int_0^\infty \int_0^{2\pi} (\zeta^L + \zeta^R) \text{Tr} \left(\rho_\sigma^{(0)R} - \rho_\sigma^{(0)L} \right) d\varepsilon d\varphi, \quad (19)$$

where

$$\zeta^l = \frac{C^l \left[(C^l)^2 - 2bk^2 \sin 2\varphi - gk^2 \right]}{(f + 2d \sin 2\varphi)^2 k^4 - 2(C^l)^2 (c + 2a \sin 2\varphi) k^2 + (C^l)^4},$$

$$C^l(U) = \Delta^l + i\frac{\hbar}{\tau},$$

$$\begin{aligned}
a &= \alpha^L \beta^L + \alpha^R \beta^R \\
b &= (\beta^L + \beta^R) (\alpha^L + \alpha^R) \\
c &= (\beta^L)^2 + (\beta^R)^2 + (\alpha^L)^2 + (\alpha^R)^2 \\
d &= \alpha^L \beta^L - \alpha^R \beta^R \\
f &= (\beta^L)^2 - (\beta^R)^2 + (\alpha^L)^2 - (\alpha^R)^2 \\
g &= (\beta^L + \beta^R)^2 + (\alpha^L + \alpha^R)^2
\end{aligned} \tag{20}$$

Parameters a - g are various combinations of the Rashba and Dresselhaus parameters of SOI in the layers. Both types of SOI are known to be small in real structures so that:

$$\alpha k_F \ll \varepsilon_F, \beta k_F \ll \varepsilon_F \tag{21}$$

This additional assumption together with (18) reduces (19) to

$$I = \frac{ie^2}{2\pi\hbar} T^2 \nu W U \int_0^{2\pi} [\zeta^L(\varepsilon_F) + \zeta^R(\varepsilon_F)] d\varphi \tag{22}$$

The integral over φ in (22) can be calculated analytically by means of complex variable integration. However, the final result for arbitrary α^l, β^l is not given here for it is rather cumbersome. In the next section some particular cases are discussed.

III. RESULTS AND DISCUSSION

The obtained general expression (22) can be simplified for a few particular important relations between Rashba and Dresselhaus contributions. These calculations reveal qualitatively different dependencies of the d.c. tunneling current on the applied voltage. The results of the cal-

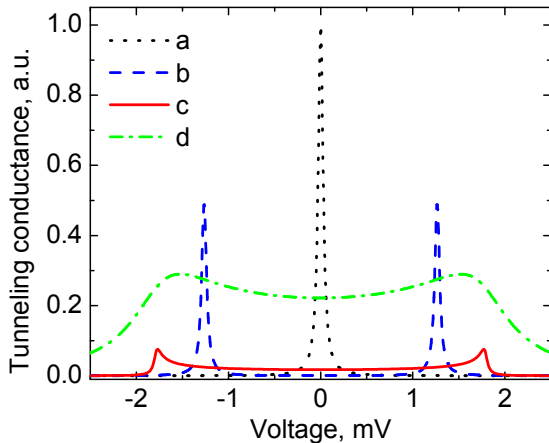


FIG. 2: Tunneling conductance, a: $\varepsilon_F = 10$ meV, $\alpha = \beta = 0$, $\tau = 2 * 10^{-11}$ s; b: same as a, but $\alpha k_F = 0.6$ meV; c: same as b, but $\beta = \alpha$; d: same as c, but $\tau = 2 * 10^{-12}$ s.

culations shown below were obtained using the following parameters: Fermi energy $\varepsilon_F = 10$ meV, spin-orbit splitting was taken to resemble *GaAs* structures: $\alpha k_F = 0.6$ meV.

A. No Spin-Orbit Interaction

In the absence of SOI ($\alpha^R = \alpha^L = 0$, $\beta^R = \beta^L = 0$) the energy spectrum for each of the layers forms a paraboloid:

$$E^l(k) = \varepsilon_0 + \frac{\hbar^2 k^2}{2m} \pm \frac{eU}{2}. \tag{23}$$

According to our assumptions (6),(7), the tunneling takes place at:

$$\begin{aligned}
E^R &= E^L \\
k^R &= k^L
\end{aligned} \tag{24}$$

Both conditions are satisfied only at $U = 0$ so that a nonzero external voltage does not produce any current despite it produces empty states in one layers aligned to the filled states in the other layer (Fig.1). The momentum conservation restriction in (24) is weakened if the electrons scatter at the impurities. Accordingly, one should expect a nonzero tunneling current within a finite voltage range in vicinity of zero. For the considered case the general formula (22) is simplified radically as all the parameters (20) have zero values. Finally we get the well-known result⁴:

$$I = 2e^2 T^2 \nu W U \frac{\frac{1}{\tau}}{(eU)^2 + (\frac{\hbar}{\tau})^2}. \tag{25}$$

The conductance defined as $G(U) = I/U$ has Lorentz-shaped peak at $U = 0$ turning into delta function at $\tau \rightarrow \infty$. This case is shown in (Fig.2,a). All the curves in Fig.2 show the results of the calculations for very weak scattering. The corresponding scattering time is taken $\tau = 2 * 10^{-11}$ s.

B. Spin-Orbit Interaction of Rashba type

The spin-orbit interaction gives qualitatively new option for the d.c. conductance to be finite at non-zero voltage. SOI splits the spectra into two subbands. Now an electron from the first subband of the left layer can tunnel to a state in a second subband of the right layer. Let us consider a particular case when only Rashba type of SOI interaction exists in the system, its magnitude being the same in both layers, i.e. $|\alpha^R| = |\alpha^L| \equiv \alpha$, $\beta^R = \beta^L = 0$. In this case the spectra splits into two paraboloid-like subbands "inserted" into each other. Fig.3 shows their cross-sections for both layers, arrows show spin orientation. By applying a certain external voltage $U_0 = \frac{2\alpha k_F}{e}$, $k_F = \frac{\sqrt{2m\varepsilon_F}}{\hbar}$ the layers can be shifted on the energy scale

in such a way that the cross-section of the "outer" subband of the right layer coincides with the "inner" subband of the left layer (see solid circles in Fig.3). At that both conditions (24) are satisfied. However, if the spin is taken into account, the interlayer transition can still remain forbidden. It happens if the appropriate spinor eigenstates involved in the transition are orthogonal. This very case occurs if $\alpha^R = \alpha^L$, consequently the conductance behavior remains the same as that without SOI. Contrary, if the Rashba terms are of the opposite signs, i.e. $\alpha^R = -\alpha^L$ the spin orientations in the "outer" subband of the right layer and the "inner" subband of the left layer are the same and the tunneling is allowed at a finite voltage but forbidden at $U = 0$. This situation, pointed out in^{9,10} should reveal itself in sharp maxima of the conductance at $U = \pm U_0$ as shown in Fig.2,b. From this dependence the value of α can be immediately extracted from the position of the peak. Evaluating (20) for this case and further the expression (22) we obtain the following result for the current:

$$I = \frac{2e^2 T^2 W \nu U \frac{\hbar}{\tau} \left[\delta^2 + e^2 U^2 + \left(\frac{\hbar}{\tau} \right)^2 \right]}{\left[(eU - \delta)^2 + \left(\frac{\hbar}{\tau} \right)^2 \right] \left[(eU + \delta)^2 + \left(\frac{\hbar}{\tau} \right)^2 \right]}, \quad (26)$$

where $\delta = 2\alpha k_F$. The result is in agreement with that derived in¹⁰, taken for uncorrelated spatial arrangement of the impurities. As we have already noted we do not take into account interlayer correlator $\langle B_{kk'} \rangle$ (13) because parametrically it has higher order of tunneling overlap integral t than the intralayer correlator $\langle A_{kk'} \rangle$. Therefore the result (26) is valid for arbitrary degree of correlation in spatial distribution of the impurities in the system. It is worth noting that the opposite case when

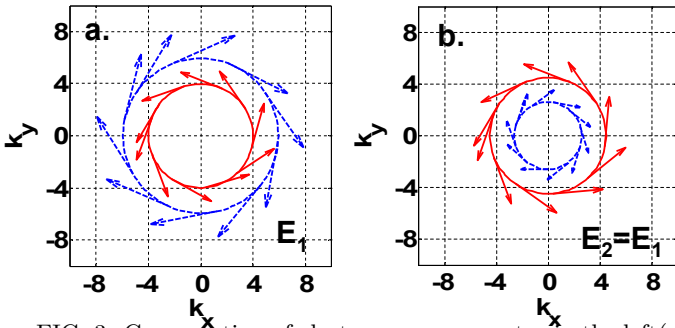


FIG. 3: Cross-section of electron energy spectra in the left (a) and right (b) layer for the case $\alpha^L = -\alpha^R, \beta^L = \beta^R = 0$.

only Dresselhaus type of SOI exists in the system leads to the same results. However, it is rather non-practical to study the case of the different Dresselhaus parameters in the layers because this type of SOI originates from the crystallographic asymmetry and therefore cannot be varied if the structure composition is fixed. For this case to be realized one needs to make the two layers of different materials.

C. Both Rashba and Dresselhaus contributions

The presence of Dresselhaus term in addition to the Rashba interaction can further modify the tunneling conductance in a non-trivial way. A special case occurs if the magnitude of the Dresselhaus term is comparable to that of the Rashba term. We shall always assume the Dresselhaus contribution being the same in both layers: $\beta^L = \beta^R \equiv \beta$. Let us add the Dresselhaus contribution to the previously discussed case so that $\alpha^L = -\alpha^R \equiv \alpha$, $\alpha = \beta$. The corresponding energy spectra and spin orientations are shown in Fig.4. Note that while the spin orientations in the initial and final states are orthogonal for any transition between the layers, the spinor eigenstates are not, so that the transitions are allowed whenever the momentum and energy conservation requirement (24) is fulfilled. It can be also clearly seen from Fig.4 that the condition (24), meaning overlap of the cross-sections a. and b. occurs only at few points. This is unlike the previously discussed case where the overlapping occurred within the whole circular cross-section shown by solid lines in Fig.3. One should naturally expect the conductance for the case presently discussed to be substantially lower. Using (22) we arrive at a rather cumbersome expression for the current:

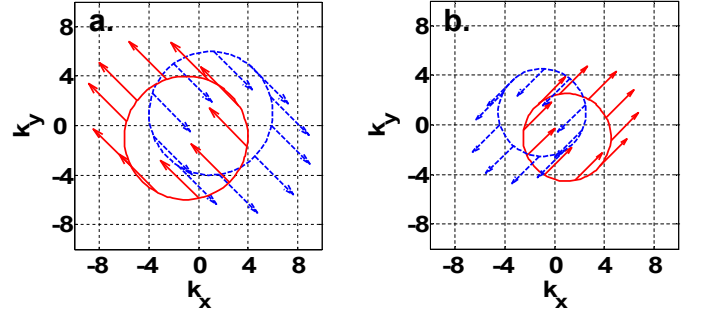


FIG. 4: Cross-section of electron energy spectra in the left (a) and right (b) layer for the case $\alpha^R = -\alpha^L = \beta$.

$$I = eT^2 W \nu U \left[\frac{G_- (G_-^2 - \delta^2)}{\sqrt{F_- (\delta^4 + F_-)}} - \frac{G_+ (G_+^2 - \delta^2)}{\sqrt{F_+ (\delta^4 + F_+)}} \right] \quad (27)$$

where

$$G_{\pm} = eU \pm i \frac{\hbar}{\tau}$$

$$F_{\pm} = G_{\pm}^2 (G_{\pm}^2 - 2\delta^2).$$

Alternatively, for the case of no interaction with impurities a precise formula for the transition rate between the layers can be obtained by means of Fermi's golden rule. We obtained the following expression for the current:

$$I = \frac{2\pi e T^2 W}{\hbar \alpha^2} \left(\sqrt{K + \frac{8m\alpha^2 eU}{\hbar^2}} - \sqrt{K - \frac{8m\alpha^2 eU}{\hbar^2}} \right), \quad (28)$$

where

$$K = 2\delta^2 - e^2 U^2 + \frac{16m^2\alpha^4}{\hbar^4}$$

Comparing the results obtained from (27) and (28) is an additional test for the correctness of (27). Both dependencies are presented in Fig.5 and show a good match. The same dependence of conductance on voltage is shown in Fig.2,c. As can be clearly seen in the figure the conductance is indeed substantially suppressed in the whole voltage range. This is qualitatively different from all previously mentioned cases. Furthermore, the role of the scattering at impurities appears to be different as well. For the considered above cases characterized by resonance behavior of the conductance, the scattering broadens the resonances into Lorentz-shape peaks with the characteristic width $\delta = \hbar/(e\tau)$. Contrary, for the last case the weakening of momentum conservation, caused by the scattering, increases the conductivity and restores the manifestation of SOI in its dependence on voltage. Fig.2,d shows this dependence for a shorter scattering time $\tau = 2 * 10^{-12}$. The reason for that is the weakening of the momentum conservation requirement due to the elastic scattering. One should now consider the overlap of the spectra cross-sections the circles in Fig.4 having a certain thickness proportional to τ^{-1} . This increases the number of points at which the overlap occurs and, consequently, the value of the tunneling current. As the calculations show, for arbitrary α and β the dependence of conductance on voltage can exhibit various complicated shapes with a number of maxima, being very sensitive to the relation between the two contributions. The origin of such a sensitivity is the interference of the angular dependencies of the spinor eigenstates in the layers. A few examples of such interference are shown in Fig.6, a–c. All the dependencies shown were calculated for the scattering time $\tau = 2 * 10^{-12}$ s. Fig.6,a summarizes the results for all previously discussed cases of SOI parameters, i.e. no SOI (curve 1), the case $\alpha_R = -\alpha_L, \beta = 0$ (curve 2) and $\alpha_R = -\alpha_L = \beta$ (curve 3). Following the magnitude of τ all the resonances are broadened compared to that shown in Fig.2. Fig.6,b (curve 2) demonstrates the conductance calculated for the case $\alpha_L = -\frac{1}{2}\alpha_R = \beta$, Fig.6,c (curve 2) – for the case $\alpha_L = \frac{1}{2}\alpha_R = \beta$. The curve 1 corresponding to the case of no SOI is also shown in all the figures for reference. Despite of a significant scattering parameter all the patterns shown in Fig.6 remain very distinctive. That means that in principle the relation between the Rashba and Dresselhaus contributions to SOI can be extracted merely from the I-V curve measured in a proper tunneling experiment.

IV. SUMMARY

As we have shown, in the system of two 2D electron layers separated by a potential barrier SOI can reveal itself in the tunneling current. The difference in spin structure

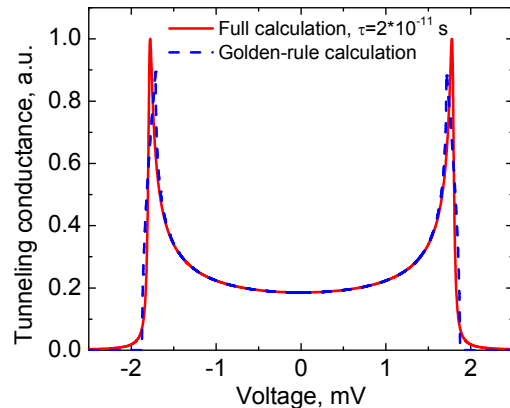


FIG. 5: Tunneling conductance calculated for the case $\alpha^R = -\alpha^L = \beta$ and very weak scattering compared to the precise result obtained through Fermi's golden rule calculation.

of eigenstates in the layers results in a sort of interference in the tunneling conductance. The dependence of tunneling conductance on voltage appears to be very sensitive to the parameters of SOI. Thus, we propose a way to extract the parameters of SOI and, in particular, the relation between Rashba and Dresselhaus contributions in the tunneling experiment. We emphasize that unlike many other spin-related experiments the manifestation of SOI studied in this paper should be observed without external magnetic field. Our calculations show that the interference picture may be well resolved for GaAs samples with the scattering times down to $\sim 10^{-12}$ s, in some special cases the scattering even restores the traces of SOI otherwise not seen due to destructive interference.

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APPENDIX

In this section we discuss an approach to the calculation of the tunneling current based on Green's function formalism. As we shall see this approach used in^{4,10} gives the same results as that based on the operator motion equation (8). However, one should be accurate with averaging over spatial distribution of the impurities. The starting point for this calculation is the tunneling Hamiltonian (1) in which the coupling between the two layers sits merely in the tunneling term H_T (2). Whenever such Hamiltonian is assumed, its part connected to the scattering at the impurities is the following

$$H_V = H_V^L + H_V^R = \sum_{k,k',\sigma} V_{kk'}^L c_{k\sigma}^{L+} c_{k'\sigma}^L + \sum_{k,k',\sigma} V_{kk'}^R c_{k\sigma}^{R+} c_{k'\sigma}^R, \quad (\text{A.1})$$

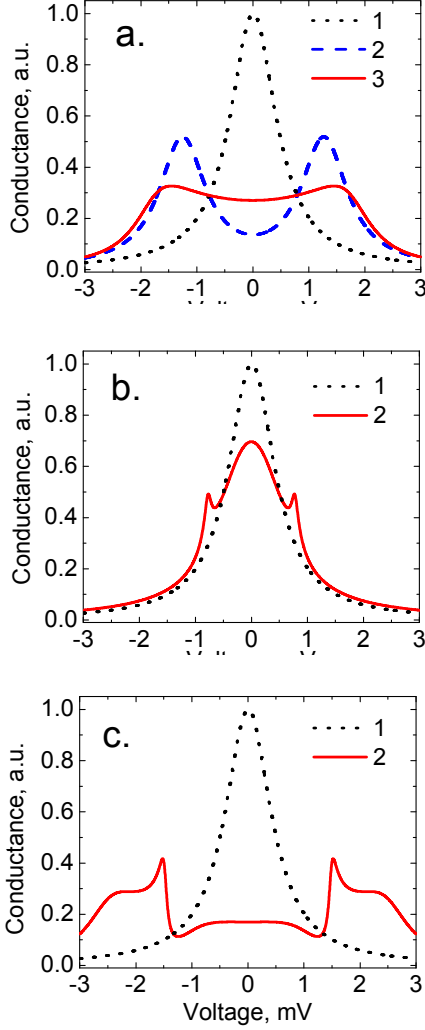


FIG. 6: Tunneling conductance calculated for various parameters of SOI

where $V_{kk'}^{ll'}$ are the matrix elements of the scattering operator calculated on the electrons eigenfunctions. Note that this Hamiltonian does not contain interlayer matrix elements V^{LR} , V^{RL} , which would have appeared after straightforward secondary quantization of the impurities external field (14). This is reasonable because these elements are parametrically small compared to intralayer elements V^L , V^R and also to the tunneling term H_T . To prove the first we recall from (16) that $V^{ll'} \sim \sqrt{B}$ while the intra-layer matrix elements $V^l \sim \sqrt{A}$. Taking into account (15) we have:

$$\frac{V^{ll'}}{V^l} \sim t \ll 1$$

The interlayer matrix elements are also parametrically small compared to the tunneling term in the Hamiltonian. Indeed, from (17),(15) also follows that $V^{ll'} \sim t \frac{\hbar}{\tau}$

($l \neq l'$), while as was mentioned earlier $T \sim t\varepsilon_F$, hence:

$$\frac{V^{ll'}}{T} \sim \frac{\hbar}{\varepsilon_F \tau} \ll 1$$

The Hamiltonian (1) can be rewritten as:

$$H = H_0 + H_V + H_T \quad (\text{A.2})$$

The calculation of the current can be carried out by means of perturbation theory with H_V, H_T treated as two perturbations. At first step only the tunneling term H_T is considered as a perturbation while the rest of the Hamiltonian describes the system treated as unperturbed. The tunneling constant T is a small parameter as soon as our consideration is restricted to the case of weak tunneling. The d.c. tunneling current can be calculated adjusting the Kubo formula to the tunneling current¹². The Kubo formula treats the perturbation to the leading order so that the leading order for the current is T^2 . We obtain the following formula for the d.c. tunneling current expressed through the unperturbed Green's functions:

$$I = \frac{eT^2W}{\hbar^3} \text{Im} \left\{ \text{Tr} \int G_{0V}^R(p, p, \varepsilon - eU) G_{0V}^L(p, p, \varepsilon) d\mathbf{p} d\varepsilon \right\}, \quad (\text{A.3})$$

where p is electron lateral momentum, G_{0V}^R, G_{0V}^L are the components of the unperturbed Green's functions (which are matrices in our case). Index V tells that at this stage, while only H_T is considered as a perturbation, these unperturbed Green's functions include the scattering by impurities. At the second step the Green's functions G_{0V}^l are to be expressed in terms of the known unperturbed Green's functions G_0^l of the non-interacting 2D electron gas. The conventional perturbation theory leads to the summation of diagrams of the two types

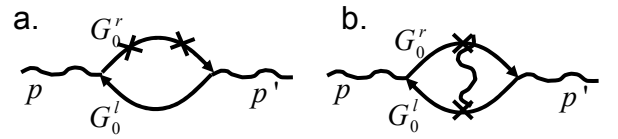


FIG. 7: Two types of diagrams for calculation of the tunneling current

turbed Green's functions G_0^l while the crosses correspond to the matrix elements $V_{kk'}^l$. The ladder diagrams of type b. give vertex corrections discussed in^{4,10}. However, in our opinion, the ladder diagrams do not give any significant impact to the tunneling current. It can be easily shown that if the interaction part of the Hamiltonian is as given by (A.1), the upper branches of the diagrams in Fig.7 (corresponding to G_0^R) contain only $V_{kk'}^R$ while the lower branches have only $V_{kk'}^L$. Hence the diagrams of type a. contain the quadratic forms of the intralayer matrix elements $A_{kk'}$ (13) while the diagrams of type b. contain only interlayer elements $B_{kk'}$. It follows from

(15) that after averaging over the impurities (with any degree of their spatial correlation) the interlayer matrix elements give higher order with regard to the tunneling

parameter T and therefore must be omitted in the leading order calculations.

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